## Dicey

By Editor Test Wed, May 22, 2013
"We have been trained to consider all the possible future outcomes [of a gamble] and then weight them in accordance with their probability. In effect we freeze time and take multiple copies of the world and then run the six versions forward as 'parallel universes'," writes this TowersWatson analyst. But the real world, he says, doesn't work that way.

As strange as it may appear, much of finance and economics implicitly assumes we have infinite lives all running in parallel. To illustrate the point, consider the following gamble.

## Experiment 1

You will roll a fair die and if you roll any number from one to five I will pay you $50 \%$ of your current wealth, including the present value of your future earnings. This is a thought experiment so we will gloss over my ability to pay - assume my credit is pristine. Imagine how much better your life would be if you were one-and-a-half times richer in the time it took to roll a die. The downside, paltry in comparison, is that if you roll a six you will pay me your entire wealth - house, pension pot, all future earnings, the lot. Will you take the gamble?

The way we have been trained to analyze the gamble means that we will consider all the possible future outcomes and then weight them in accordance with their probability. In effect we freeze time and take multiple copies of the world and then run the six versions forward as 'parallel universes'.

In one of those worlds a one is rolled and we pocket a $50 \%$ gain in wealth. In the second a two is rolled with the same result. In the sixth world a six is rolled and we lose all our wealth. Having exhausted all the possibilities we travel back in time to the present and do our sums.

The expected return of the gamble is the ensemble average - the average of all the possible independent outcomes. In this case the expected return is $25 \%$ and so we would be 'crazy' not to take it.

So would you take the gamble? The answer is typically 'no'.

Instinctively, something doesn't feel right. Either you don't trust my credit, or the ensemble average (expected return or expected value) is misleading in some way. So let's consider the time average instead. Instead of rolling the die once in each of six parallel universes, we will stay in our familiar universe and roll the same die six times in succession.

We compute the time average by taking each of the six possible independent outcomes and making them occur one after the other in our single, real, universe. We now compound our returns over the six periods and take the sixth-root to calculate our per-period expected (time average) return. It doesn't matter what order we roll each of the numbers one to six, we will lose all our wealth and so the time average is negative, and in a big way (minus-100\%).

So the ensemble average is misleading. The $25 \%$ expected return unhelpfully disguises the meaningful (16.7\%) likelihood that we lose everything.

The point of this thought experiment is to introduce the notion that we cannot go backwards in time, as once we have lost everything we can't go back and try again. The more subtle point is that the traditional calculation we use, the expected return (ensemble average), effectively under-weights the significance of extreme risks.

## Experiment 2

We can illustrate this with a more realistic thought experiment. Consider a world, in which we are investing our portfolio of financial assets, where there are two types of outcomes; good outcomes, which produce a return of five percent and occur almost all the time, and extreme outcomes which only occur once in the distribution but cause severe or total loss.

If the ensemble average truly does understate the significance of the single extreme event then it will consistently overestimate the likely return our portfolio will achieve relative to the return the time average suggests we will achieve.

Table 2 shows that this is indeed the case for a number of 'runs' of our thought experiment world. The pairs of columns represent runs for worlds with different probabilities for the single extreme outcome starting with one-in-1000 and moving right to one-in-100. The rows also show different worlds, where the severity of portfolio loss increases from $99 \%$ in the first row to $100 \%$ in the bottom row.

Note that for all 20 runs (combinations of probability and severity) the ensemble average return is always higher than the time average return, and in some cases significantly higher. In addition, please note that once the probability of the extreme event gets up to one-in-100, or higher, then 99 good outcomes of five percent are wiped out by a single extreme event.

Finally, note the difference in the time average return between extreme losses of 99.999\% and 100\%. Like our artificial die throwing experiment above, once you have lost all your wealth the game is over and your return is minus-100\%, whether that occurs in the first period or the last period. Losing $99.999 \%$ of your portfolio would clearly be painful, but the little that is left can then start to grow again. Essentially this is highlighting the difference between an existential risk and a risk where 'life' continues into the next period, albeit in very poor shape.

Table 2 here

Now, it is possible to object that a loss on a portfolio of $99 \%$ or more is too extreme to realistically contemplate. The point of a portfolio, after all, is to diversify against such extreme losses. Clearly this is a valid objection; however this is a thought experiment so the value is in what it teaches rather than the realism.

That said, I believe that some of the extreme risks that we discuss below* could indeed cause portfolios of
financial assets to become worthless. Besides, there are several historical examples of entire stock market losses.

Returning to the learning point though, rather than consider a literal 99\% loss (or greater) instead consider what effect a large portfolio loss could have on a retirement fund. Here you can mentally adjust 'large' as you see fit, but perhaps start with a $50 \%$ loss and adjust higher and lower.

If the retirement fund is a defined benefit arrangement and the sponsoring employer is now small relative to the fund, or has ceased trading, then I would argue that a large, and feasible, portfolio loss can represent an existential event in that context. By 'existential' in this instance I mean that the mission of the retirement fund will have failed at that point. The assets will run out before the liabilities are paid and, absent an insurance arrangement, some of the beneficiaries will receive nothing. So for them at least this would equate to a total portfolio loss.

If instead the retirement fund was a defined contribution arrangement then it is less likely that the large portfolio loss would qualify as existential. This is due to the fact that there is no contractual benefit to be broken. Instead the members 'get what they get' and the adjustment mechanism is up to the individual member, perhaps by accepting a lower standard of living in retirement than hoped for. Even here, however, not all members are equal.

A $50 \%$ loss for a 29 -year-old is fundamentally different to a $50 \%$ loss for a 59 -year-old, and the older member may be tempted to consider that they had suffered a loss bordering on the existential. The practical takeaway is that avoiding, or reducing the probability of $100 \%$ (existential) losses is incredibly valuable and should become a top priority.

I am arguing that extreme risks matter, and deserve more attention than they have been given thus far. In addition, the global economic environment continues to be characterized by significant imbalances and consequently is not in good shape to withstand any further major shocks.

Partly as a consequence of economic conditions and the ever-present risk of extreme events, but also due to very nature of the retirement system and Keynes's paradox of thrift, I believe that retirement for the masses (as currently configured in terms of length, quality of life, and degree of financial freedom) is at serious risk.

Or, rather, that retirement as currently configured was never affordable, but this fact was hidden by demographic and debt trends over many decades. Serious collective discussion and actions could lessen this risk to retirement for the benefit of all-albeit that the most obvious action is to lower the general level of expectations concerning retirement.
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