

How to Demonstrate the Value of an Income Annuity

By Editor Test Mon, Aug 15, 2011

In this edited version of a monthly bulletin from the Qwema Group, Moshe Milevsky (left) and Simon Dabrowski show advisors how to demonstrate the value of an income annuity to their clients.

$$PV(C, R, D) = C \frac{1 - e^{-R(D-Age)}}{R}$$

Note: This Q&A has been condensed. A complete version is available [here](#).

Question of the Month: My clients have unrealistic expectations regarding how much they can spend during retirement, given the size of their nest eggs. It’s just not enough. What is the best way to explain this to them? How sustainable is their retirement income plan if they do not reduce their standard of living?

Answer:

Let us start with some basic retirement arithmetic. Imagine your client is exactly 65 years old and he/she would like to retire today. Besides the entitled income from government and corporate pensions, they have determined they need an additional cash flow of \$1,000 per month (\$12K per year) for the rest of their life. We will assume that these monthly desires are expressed in real inflation-adjusted terms (i.e. today’s dollars).

So, how much of a lump sum do they need today, to generate this specified stream of income for the rest of their life? In order to calculate the required size of the nest egg, we have applied the following formula. This function describes the present value of a term certain (non-random) annuity discounted at the appropriate rate in continuous time.

$$PV(C, R, D) = C \frac{1 - e^{-R(D-Age)}}{R}$$

In this case, ‘C’ represents, consumption or \$12,000, ‘R’ represents the appropriate real interest rate, ‘D’ represents the number of years in retirement and ‘Age’ represents the age of your client.

Table 1 provides values assuming investment returns of 0%, 1.5%, 4.0% and 6.5% and income plans that last to ages 84, 90 and 97. (We have selected these odd-looking numbers deliberately, for reasons that will soon be clear.) Also within Table 1, for comparison, is an estimate for the cost of a \$1,000 per month life annuity, purchased at the age of 65.

Table 1

What Size Nest Egg Do You Need?					
Retiring at Age 65 and Requiring \$1,000 of Monthly Income for Life					
	REAL Inflation-adjusted Investment Return (Interest Rate)				
	Age	0.0%	1.5%	4.0%	6.5%
Invest & Plan to Life Expectancy:	84.2	\$230,500	\$200,300	\$160,900	\$131,600
Invest & Plan to 75th Percentile:	90.1	\$301,700	\$251,300	\$190,300	\$148,600
Invest & Plan to 95th Percentile:	97.1	\$385,100	\$305,700	\$216,900	\$161,700
Purchase (REAL) Life Annuity:		N.A.	\$236,900	N.A.	N.A.

Here is how to read and interpret Table 1. If you are retiring at the age of 65 and would like an income stream until life expectancy, which is age 84.2—after which, we presume, you plan to shoot yourself—and this money is invested at a rate of 1.5%, then you will need a nest egg of a little over \$200,000 at retirement. So says the math.

We deliberately selected 1.5% as the investment return in the above paragraph, since it is the best rate you can guarantee in today's environment on an after-inflation basis. Note that in late July 2011, long-term inflation-linked (U.S. Government) bonds yielded 1.5%. We all might believe this is artificially low, but it is the best you can get if you want something that is guaranteed.

If you plan your retirement to the 75th percentile of the mortality table, which is age 90, then you need a retirement nest egg of approximately \$251,000. The extra \$51,000 will generate the \$1,000 monthly income for the extra six years. Stated differently, the present value of \$1,000 per month until the age of 90 is \$251,000 when discounted at 1.5%. If you plan to live to age 97, then you need a nest egg of approximately \$306,000 to generate the \$1,000 of monthly income.

This is a basic application of the time value of money, given today's interest rates. Of course, most people look at the \$306,000 price tag for a meager \$1,000 and balk, or they get very depressed. Scale this up by a factor of 10, for those who want a monthly income of \$10,000 and retirement will cost a cool \$3 million, if you want the money to last to the age of 97—which you have a 5% (one in 20) chance of reaching.

Enter the retirement planning software used by confused (or unscrupulous) advisors and they have a better and more soothing answer. If you invest more aggressively (that software will tell you) then you won't need to assume the small, pathetic and depressing 1.5% real return in the above table. If (the advisor might say) you purchase more equity-based mutual funds, or invest more heavily in stocks, then you can use the much higher 6.5% column. Why? *"Because in the long run, stocks have averaged 6.5% after inflation, even if you include the fees I will be charging."*

So, the story often goes, "If you take a bit more equity market risk, all you need is \$131,600 at retirement if

you plan to life expectancy. And, even if your retirement horizon is age 90, then all you need is \$148,600 at retirement, per \$1,000 of monthly income. As for age 97, don't worry about it (they say). Most people don't reach that age."

We believe this is the wrong approach. Assuming a more aggressive portfolio, in the hopes that you can move to the upper right-hand corner of Table 1—and hence require a smaller nest egg for retirement—is a mirage. You can't tweak expected return (a.k.a. asset allocations) assumptions until you get the numbers that you like.

Very low real interest rates, such as 1.5% currently available, translate into a high cost of retirement, and vice versa. Betting that these rates will eventually go back to normal, or that equity markets will make your retirement cheaper, is just that—betting. In fact, this sort of thinking is precisely the mistake that got the pension fund industry (and many of their actuaries) into big trouble.

Here is one of the axioms of financial economics. If you are going to assume a higher expected investment return—like 6.5%—compared to what is available with no risk, then you must also allow for the possibility that things will not work out and you might earn much less than expected. Average the two scenarios—and account for this risk properly—and you are left exactly where you started, namely the present value of your \$1,000 under a risk-free return is \$230,500 if you plan to life expectancy and \$385,100 if you plan to the 95th percentile.

If you do not like how big this number looks—and you want certainty—then save more, retire later and plan to spend less. Assuming, expecting or anticipating 6.5% and/or planning to die at age 90 won't solve a structural funding problem. Greece is a nice place to retire, but not a very good role model for how to manage retirement finances.

Now let us get to the second of two points, which is the estimate for the cost of a real inflation-adjusted life annuity, displayed in the final row of the table.

If you spend \$236,900 on a life annuity from an insurance company, it will generate the desired \$1,000 per month income—adjusted by the consumer price index—with no investment or mortality risk (that is, no chance that your income will drop or you will run out of money before you die). You do not have to *assume* how long you will live or *assume* what your portfolio will earn over the random horizon of retirement.

As such, the \$236,900 is effectively the cost of your retirement income plan. Any other answer involves extra risk, possibly invisible to the naked eye. It is often obscured from view due to heroic assumptions hardwired into some retirement planning calculators.

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